FMDB ***Transactions on Sustainable Technoprise Letters***

Dynamics of Equilibrium and Long-Term Volatility of the Crude Oil Prices: ECM and VECH-GARCH Approach

Chinwe Roseline Okoye[1,*], I. D. Essi[2]

[1,2]Department of Mathematics, Rivers State University, Port Harcourt, Rivers State, Nigeria.
rakpu4real@yahoo.co.uk[1], caser.ph@gmail.com[2]

Abstract: West Texas Intermediate and Dubai/Oman crude oil price parameters. Brent and Dubai/Oman crude oil prices exhibit significant negative error-correction rates (-0.507 and -0.821), indicating a strong tendency to return to long-term equilibrium after short-term shocks. In contrast, West Texas Intermediate (WTI) shows a positive, though statistically insignificant, coefficient, suggesting divergence and poor integration with global oil prices. The diagnostic tests showed Brent and Dubai/Oman crude oil price inequality and non-normality, justifying the use of the VECH-GARCH model. VECH-GARCH estimates that exogenous shocks influence oil price swings (ARCH effects ranging from 0.026 to 0.105) and show strong stability (GARCH coefficients up to 0.965). Portmanteau and Q-Q diagnostic tests show that the model reflects conditional variance behaviour. Brent and Dubai/Oman adapt better to global market signals than West Texas Intermediate, which is distorted by regional factors. These findings highlight the need to select criteria, integrate volatility modelling into decision-making, and implement region-specific market reforms for market participants, policymakers, and risk managers. Joint modelling of equilibrium correction and volatility transition provides a viable framework for understanding global oil pricing dynamics.

Keywords: West Texas Intermediate; Brent Crude Oil Prices; VECH-GARCH Model; Dynamics and Equilibrium; Error Correction Models; Multivariate GARCH Models; Augmented Dickey-Fuller; Phillip Perron Test.

Received on: 29/01/2025, **Revised on:** 30/04/2025, **Accepted on:** 25/07/2025, **Published on:** 23/11/2025

Journal Homepage: https://www.fmdbpub.com/user/journals/details/FTSTPL

DOI: https://doi.org/10.69888/FTSTPL.2025.000505

Cite as: C. R. Okoye and I. D. Essi, "Dynamics of Equilibrium and Long-Term Volatility of the Crude Oil Prices: ECM and VECH-GARCH Approach," *FMDB Transactions on Sustainable Technoprise Letters*, vol. 3, no. 4, pp. 200–210, 2025.

1. Introduction

The Nigerian economy remains heavily dependent on crude oil, which constitutes the primary source of government revenue and export earnings. However, the market for Nigerian crude oil is increasingly plagued by high price volatility, inconsistent pricing behaviour, and weak transmission mechanisms to the broader financial and economic systems. This volatility, driven by global factors such as geopolitical tensions, OPEC+ decisions, and shifts in global demand, has introduced substantial uncertainty into Nigeria's macroeconomic environment [3]. Compounding this instability are the country's limited economic diversification and its underdeveloped financial system's capacity to absorb external shocks. Recent inconsistencies in price movements across major crude oil benchmarks such as Brent, WTI, and Dubai/Oman have raised concerns about the efficiency and effectiveness of Nigeria's price discovery processes. These discrepancies impair policymakers, investors, and risk

*Corresponding author.

managers' ability to anticipate and respond to oil market shocks, thereby limiting the efficacy of monetary and fiscal interventions. Furthermore, there is a dearth of empirical research that employs advanced econometric models, such as Error Correction Models (ECMs) and Multivariate GARCH models, to rigorously analyse these dynamics and their macroeconomic implications [11].

Simultaneously, Nigeria faces significant volatility in key macroeconomic indicators, including exchange rates, central bank interest rates, inflation, real GDP, and labour market variables, suggesting structural imbalances, policy inconsistencies, and external vulnerabilities [14]. These irregularities not only challenge the effectiveness of monetary policy but also diminish investor confidence and economic predictability [12]. Moreover, empirical features of the data, such as non-normality and the need for differencing to achieve stationarity, suggest the presence of underlying shocks and delayed policy effects that traditional macroeconomic models fail to capture adequately [15]. Structural estimations from MGARCH models further reveal that certain shocks, particularly those linked to government spending and technology, have persistent impacts on output and inflation. In contrast, others are transitory, thereby complicating both short-term stabilisation and long-term policy planning [13]. Considering these complexities, there is a pressing need to investigate the dynamics, transmission mechanisms, and persistence of monetary policy shocks in Nigeria [16]. A deeper understanding of oil price behaviour and its interaction with macroeconomic variables is crucial for designing more robust and responsive policy tools to enhance economic stability, foster sustainable growth, and improve macroeconomic governance.

2. Methodology

A time plot provides a graphical representation of a data set, with the x-axis representing time and the y-axis the variable being measured [7]. This visualisation helps to identify trends, seasonality, and cycles within the series. It is useful for recognising upward or downward trends over time; a rising line suggests increasing values, while a declining line indicates decreasing values. Seasonal patterns may also be visible in a time plot as repeated fluctuations at regular intervals. For example, if you plot monthly sales, you might see peaks every December. Also, the normality test is carried out using the Jarque-Bera test statistic. According to Ewing and Malik [6], the Jarque-Bera test is a joint test of skewness and kurtosis that examines whether a data series exhibits normality; it was developed by Su et al. [12]. It is defined as.

$$X^2_{\simeq} \quad \frac{N}{6}\left[S^2 + \frac{(K-3)^2}{4}\right] \tag{1}$$

Where S represents Skewness, K represents Kurtosis, and N represents the size of the macroeconomic variables used. The test statistics under the null hypothesis of a normal distribution have 2 degrees of freedom 2. When a distribution does not meet the normality assumption, Bollerslev et al. [2] suggested using multivariate GARCH with error distributions assumed to have fixed degrees of freedom. Similarly**,** the Unit Root Test for stationarity is conducted using the Augmented Dickey-Fuller (ADF) test, which is commonly employed in time-series analysis to determine the order of integration of a series. Unit root test **is** very vital in time series analysis, and this will be done using the Augmented Dickey-Fuller (ADF) and Phillips-Perron Test (PPT). The unit root test assumes that a series follows a random walk.

$$Y_t = b_1 y_{t-1} + \varepsilon_t, \text{ Random walk} \tag{2}$$

$$Y_t = b_0 + b_1 y_{t-1} + \varepsilon_t, \text{ Random walk with drift} \tag{3}$$

$$Y_t = b_0 + b_1 y_{t-1} + b_2 t + \varepsilon_t, \text{ Random walk with drift and trend} \tag{4}$$

However, to enhance stationarity, we considered whether y_{t-1} is subtracted from the Right Hand Side (RHS) of each of equations 3.3 -3.17, we have;

$$Y_t - Y_{t-1} = b_1 Y_{t-1} - Y_{t-1} + \varepsilon_t, \ \Delta Y_t = \vartheta Y_{t-1} + \varepsilon_t, \text{ Random walk} \tag{5}$$

$$Y_t - Y_{t-1} = b_0 + b_1 Y_{t-1} - Y_{t-1} + \varepsilon_t, \ \Delta Y_t = b_0 + \vartheta Y_{t-1} + \varepsilon_t, \text{ Random walk with drift} \tag{6}$$

$$Y_t - Y_{t-1} = b_0 + b_1 Y_{t-1} - Y_{t-1} + b_2 t + \varepsilon_t, \Delta Y_t = b_0 + \vartheta Y_{t-1} + b_2 t + \varepsilon_t, \text{ Random walk with drift and trend}$$

Where, $b_1 Y_{t-1} - Y_{t-1} = (b_1 - 1)Y_{t-1}$, let $(b_1 - 1) = \vartheta$, we have ϑY_{t-1} and $Y_t - Y_{t-1} = \Delta Y_t$

2.1. The Null Hypothesis is tested as follows:

For a pure random walk, we have;

$$\Delta Y_t = \vartheta Y_{t-1} + \sum_{i=1}^{\rho} \sigma_i \Delta Y_{t-1} + \varepsilon_t \quad (7)$$

- **H0:** $\vartheta = 0$ and therefore r = 1 against the alternative that **HO₁:** $\vartheta < 0$ and r < 1. Similarly, a Random walk with drift, we have

$$\Delta Y_t = b_0 + \vartheta Y_{t-1} + \sum_{i=1}^{\rho} \sigma_i \Delta Y_{t-1} + \varepsilon_t \quad (8)$$

- **H0:** $\vartheta = 0$ and therefore r = 1 against the alternative that **HO₁:** $\vartheta < 0$ and r < 1. Also, a random walk with drift and trend.

$$\Delta Y_t = b_0 + \vartheta Y_{t-1} + \sum_{i=1}^{\rho} \sigma_i \Delta Y_{t-1} + b_2 t + \varepsilon_t \quad (9)$$

- **H0:** $\vartheta = 0$ and therefore r = 1 against the alternative that **HO₁:** $\vartheta < 0$ and r < 1.

The decision that follows will be considered if '$Y_t$it is found to be more negative and statistically significant at the 5 per cent level. We compare the parameter's t-statistic with the tabulated critical value. We reject the null hypothesis and accept the alternative, concluding that the series does not have a unit root. Conversely, if we accept the null hypothesis and reject the alternative, we conclude that the series has a unit root. After that, we determine the lag length. Differences refer to situations in which differencing is required to obtain stationarity. If the series is expressed as an AR process and the AR polynomial contains a unit root, that is, if one root of the autoregressive polynomial lies on the unit circle, e.g., for an AR(1), $\alpha = 1$, then differencing is necessary. According to Smyth and Narayan [13], this test is used to check if the ε_t obtained in the resulting model (3.9), the residuals violate the assumption of homoskedasticity and therefore, the regression is given as thus:

$$\varepsilon_t^2 \ = \propto_0 + \alpha_1 \varepsilon_{t+}^2 + . \ \ . \ \ \dots\dots\dots\dots + \alpha_p \varepsilon_{t-p}^2 + \mu_\zeta \quad (10)$$

where $\alpha_1 \dots\dots\dots\dots\dots\dots\dots\dots \alpha_p$ are the coefficients of the regression and $\alpha_0$is considered the intercept of the ARCH model specified in (3.14). The hypothesis of the ARCH effect is given as:

- **H₀:** $\alpha_1 = \alpha_2 = \dots\dots\dots\dots\dots\dots = \alpha_p = o$, there is no ARCH effect in the residuals under the null of the lagrange multiplier (LM) statistic is distributed asymptotically as $x^2(p)$ statistic against the alternative hypothesis.
- **H₁:** $\alpha_1 \neq 0$for some i ($\varepsilon_{1,2\dots\dots\dots q}$) at least one variable has the presence of the ARCH effect.

Deebom and Essi [3] noted that the number of observations times the R-squared (nR^2) gives the joint significance of the test statistics with the q- lagged squared residuals with q degrees of freedom and its estimated probability value. Deebom and Essi [3] further explained that nR^2 is tested against the $X^2(q)$ distribution, meaning that if nR^2> (q), the result in the Table, then the null hypothesis will be rejected, and it is concluded that an ARCH effect is present. Conversely, where $nR^2 < X^2(q)$ is based on the results in the Table, then there is the absence of an ARCH effect in the residuals obtained from the ARMA model. The Vector Error Conditional Heteroscedasticity (VECH)-GARCH models allow the conditional covariance matrix of the dependent variables to follow an elastic dynamic structure [2]. In the case of the VECH, the conditional variance and covariance would each depend upon lagged values of all the variances and covariances and on lags of the squares of both error terms and their cross products. Suppose that there are four variables used in the model. The conditional covariance matrix is denoted H_t, and this would be 3 (3). H_t and VECH (H_t) are written in matrix form as follows:

$$\sigma_{i,t}^2 = M(i) + A1(i) * \varepsilon_{1,t-1}^2 * \left(\varepsilon_{1,t-1}^2\right)^1 + B1(i) * \sigma_{1,t-1}^2 \ , \ \varepsilon_t/\sigma_{t-1} \sim N(0, H_t) \quad (11)$$

where M(i), A1(i), and B1(i) are parameters of an indefinite matrix. The EViews form matrix representation of the Vector Error Conditional Heteroskedasticity (VECH)-GARCH is given as:

$$M = \begin{bmatrix} M(1,1) & M(1,2) & M(1,3) \\ & M(2,2) & M(2,3) \\ & & M(3,3) \end{bmatrix}$$

$$A = \begin{bmatrix} A1(1,1) & A1(1,2) & A1(1,3) \\ & A1(2,2) & A1(2,3) \\ & & A1(3,3) \end{bmatrix}$$

$$B = \begin{bmatrix} B1(1,1) & B1(1,2) & B1(1,3) \\ & B1(2,2) & B1(2,3) \\ & & B1(3,3) \end{bmatrix}$$

$$\sigma_t^2 = M + A1 * \varepsilon_t * \varepsilon_t^1 + B1 * \sigma_{t-1}^2$$

- M is an indefinite matrix.
- A1 is an indefinite matrix*
- B1 is an indefinite matrix*

Variance and Covariance Equations:

$$\sigma_{1,t}^2 = M(1,1) + A1(1,1) * \varepsilon_{t-1}^2 + B1(1,1) * \sigma_{t-1}^2$$
$$\sigma_{2,t}^2 = M(2,2) + A1(2,2) * \varepsilon_{t-1}^2 + B1(2,2) * \sigma_{t-1}^2$$
$$\sigma_{3,t}^2 = M(3,3) + A1(3,3) * \varepsilon_{t-1}^2 + B1(3,3) * \sigma_{t-1}^2$$
$$\rho_{1,2} = M(1,2) + A1(1,2) * \varepsilon_{1,t-1} * \varepsilon_{2,t-1} + B1(1,2) * \rho_{1,2,t-1}$$
$$\rho_{1,3} = M(1,3) + A1(1,3) * \varepsilon_{1,t-1} * \varepsilon_{3,t-1} + B1(1,3) * \rho_{1,3,t-1}$$
$$\rho_{2,3} = M(2,3) + A1(2,3) * \varepsilon_{2,t-1} * \varepsilon_{3,t-1} + B1(2,3) * \rho_{2,3,t-1}$$

According to Liu et al. [14], the most common method for estimating the conditional covariance matrix in the MGARCH model is the quasi-maximum likelihood method. Assuming Ht(θ) is a positive definite N×N conditional covariance matrix of some N×1 residual vector εt, parameterised by the vector θ. Denoting the available information at time t by f_t, we have

$$\varepsilon_{t-1}[\varepsilon_t / f_{t-1}] = 0 \qquad (12)$$

$$\varepsilon_{t-1}[\varepsilon_t \varepsilon_t^1 / f_{t-1}] = H_t(\theta) \qquad (13)$$

Generally, the conditional covariance matrix Ht(θ) is well specified based on a certain MGARCH model. Suppose there is an underlying parameter vector θ0 that one wants to estimate using a given sample of T observations. The quasi-maximum likelihood (QML) approach estimates θ0 by maximising the Gaussian log likelihood.

$$\log L_T(\theta) = \frac{-N.T}{2} \mathrm{Log}(2\Pi) - \frac{1}{2}\sum_{t=1}^{T} \mathrm{Log}/H_t/ - \frac{1}{2}\sum_{t=1}^{T} \Xi_t^1 H_t^{-1} \Xi_t \qquad (14)$$

One needs to note the assumption that the time series under consideration is stationary and that its residuals have a predefined conditional Gaussian distribution. The latter assumption can meanwhile provide us with hints on how to assess the adequacy of the established MGARCH model.

3. Results

The Preliminary Tests specified in the methodology include; time plot, the descriptive statistics for both raw and return on stock market prices, the correlation analysis between raw and returns on crude oil price benchmarks, the unit test on raw and return on stock market prices, the plots of the returns on stock market prices in three major crude oil price markets, the cointegration analysis of the returns on stock market prices using trace and maximum eigenvalue and the lagged length specifications test. The time plot of weekly crude oil prices from 1990 to 2024 shows trends for three key variables: Brent (COBRET), Dubai/Oman (COD), and WTI (COWTI).

All three exhibited an overall upward trajectory with significant volatility associated with major global events. Prices remained relatively stable in the 1990s but diverged after 2000, when WTI was consistently higher, reflecting U.S.-specific market conditions. Notable peaks occurred around 2008 and 2011-2014 due to the global financial crisis and geopolitical tensions, followed by a sharp decline in 2020 caused by the COVID-19 pandemic. Prices rose partially afterwards, albeit unevenly. The plot highlights strong inter-movements across benchmarks, the interconnectedness of the global oil market, and the influence of regional factors and macroeconomic shocks (Figure 1).

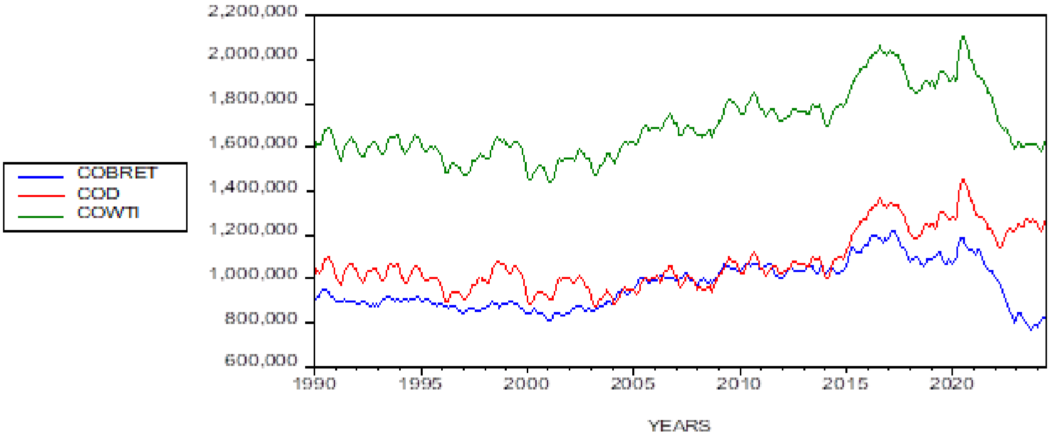

Figure 1: Time plot of weekly crude oil price in Brent blend, Dubai/Oman, and WTI

Descriptive statistics for the raw and returns of stock market prices were estimated to test the normality of the series; the results are shown in Table 1.

Table 1: Descriptive statistics of the raw and returns on stock market prices

	COBRT	WTI	COD
Mean	971	170	108.
Median	961	166	104.
Maximum	123	212.	146.
Minimum	765	144.	866
Std. Dev.	107	153	131
Skewness	0.335	0.683	0.792
Kurtosis	2.087	2.654	2.640
Jarque-Bera	96.065	148.923	197.233
Probability	0.000	0.000	0.000
Sum	1.75E+09	3.06E+09	1.94E+09
Sum Sq. Dev.	2.06E+13	4.23E+13	3.09E+13
Observations	1798	1798	1798

Table 1 contains the results of descriptive statistics of the returns and raw on weekly Crude oil prices in Brent Blend (COBRT), West Texas Intermediate(WTI), and Dubai/Oman. The mean values of the crude oil price series indicate that Brent Blend (COBRT) averaged 971, WTI (WTI) averaged 170, and Dubai/Oman (COD) averaged 108, reflecting significant price differences across the benchmarks. This implies that Dubai/Oman crude oil prices exhibit the greatest volatility, making them less stable than Brent Blend and WTI. Similarly, the standard deviation of the crude oil prices, Brent Blend (107), has the lowest price volatility, while WTI (153) and Dubai/Oman (131) exhibit greater fluctuations. For the crude oil price series, the skewness values are positive (COBRT = 0.335, WTI = 0.683, and COD = 0.792), indicating a slight rightward skew, with prices tending to experience more frequent small increases than decreases. The crude oil price series, however, exhibits lower kurtosis values (COBRT = 2.087, WTI = 2.654, and COD = 2.640), indicating distributions closer to normal, with a lower likelihood of extreme price fluctuations.

The Jarque-Bera statistics for all series are significantly high, with p-values of 0.000, indicating strong rejection of the null hypothesis of normality. This confirms that the return series does not follow a normal distribution, with implications for risk modelling and forecasting, as extreme price movements occur more frequently than predicted by standard normal-based models. The lack of normality suggests that financial models relying on normality assumptions may not adequately capture the behaviour of crude oil returns and should be adjusted accordingly. Therefore, the return series for crude oil exhibits mild negative and positive trends, with Dubai/Oman returns showing the highest volatility. The return distributions are nearly symmetric but exhibit slight excess kurtosis, leading to a higher frequency of extreme price movements. The strong rejection of normality indicates that more advanced econometric techniques, such as GARCH models or heavy-tailed distributions, may be more appropriate, as suggested by studies such as those by Deebom and Tuaneh [4]. GARCH models may be necessary for accurate volatility modelling and risk management. However, crude oil price series display greater stability in their distributions, though they still exhibit some rightward skew. Similarly, correlation analysis of the raw and returns on stock market prices was estimated, and the results are shown in Table 2.

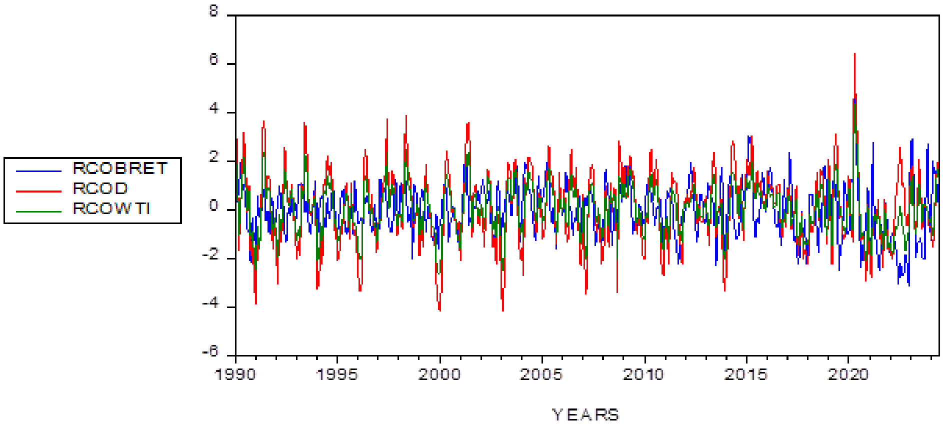

Figure 2: Time plot of weekly returns on crude oil prices, Brent Blend, West Texas Intermediate (WTI), and Dubai/Oman

Figure 2 presents a time-series plot of weekly returns for three major crude oil benchmarks. The plot enables visual comparison of volatility patterns, trends, and cyclical behaviour among Brent, WTI, and Dubai/Oman over the selected period. From Figure 2 above, it was found that volatility clustering is present in the transformed series.

Table 2: Test for cointegration of stock market prices

Unrestricted Cointegration Rank Test (Trace)					Unrestricted Cointegration Rank Test (Maximum Eigenvalue)			
Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**	Eigen Value	Max-Eigen Statistic	0.05 Critical Value	Prob.**
r= 0	0.126	458.606	29.797	0.000	0.126	241.495	21.132	0.000
r≤ 1	0.089	217.111	15.495	0.000	0.089	166.983	14.265	0.000
r≤ 2	0.028	50.129	3.841	0.000	0.028	50.129	3.841	0.000
r≤ 3	0.126	458.606	29.797	0.000	0.126	241.495	21.132	0.000

The results of the cointegration test indicate the presence of long-run equilibrium relationships among stock market prices, as shown in Table 4. The Trace test and Maximum Eigenvalue test both reject the null hypothesis of no cointegration at all levels ($r = 0$, $r \leq 1$, $r \leq 2$, and $r \leq 3$), as the computed test statistics exceed the respective critical values at the 5% significance level. At $r = 0$, the trace statistic (458.606) is far greater than the critical value (29.797), with a probability of 0.000, confirming the rejection of the null hypothesis of no cointegrating equation. Similarly, the maximum eigenvalue statistic (241.495) exceeds the critical value (21.132), further validating the existence of a long-run relationship. This pattern continues for $r \leq 1$ and $r \leq 2$, reinforcing the robustness of the cointegration findings. The presence of cointegration implies that despite short-term fluctuations, stock market prices move together over time, suggesting a stable equilibrium relationship. This result aligns with prior studies in financial markets, such as Deebom and Tuaneh [4], which demonstrate that financial and commodity markets often exhibit long-term integration despite short-run volatility. The implications of these findings are significant for investors and policymakers, as they suggest that crude oil prices in the stock market are interdependent in the long run, meaning price movements in one market could influence the other.

Table 3: Results of the error correction model

Variables	Cointegration Rank	ECT	Residuals Heteroscedasticity Test	VEC Residual Normality Tests
COBRT	3	-0.507(0.03)	543.658 F(20,000)	42.618 (df=3),(0.000)
WTI	3	0.658(0.060)	543.658 F (20, 0.415)	40.900(df=3), (0.100)
COD	3	-0.821(0.072)	543.658 F (20, 0.000)	42.618 (df=3), (0.000)

Note: *The Value in parentheses is the Degree of Freedom and p-value.*

The error correction model (ECM) results presented in Table 3 provide insight into the speed and direction of crude oil price yield adjustments – specifically Brent (COBRT), West Texas Intermediate (WTI), and Dubai/Oman (COD) – towards long-term equilibrium after short-term shocks. The error correction (ECT) expression represents the percentage of imbalance in the previous period that was corrected in the current period. For COBRT, the ECT is -0.507 with a p-value of 0.03, which is statistically significant at the 5% level. This means that about 50.7% of any imbalances from the long-term relationship will be corrected in the coming period, indicating a strong, statistically significant adjustment rate toward balance for Brent. For WTI, ECT is 0.658 with a p-value of 0.060. This coefficient is marginally positive and insignificant at the 5% level, but becomes significant at the 10% level. However, positive ECT does not make sense in the context of ECM, as it refers to divergence rather than convergence, meaning that WTI does not respond in a theoretically consistent way to deviations from long-term equilibrium, possibly due to structural or regional market factors that delay adjustment.

For COD, the ECT is -0.821 with a p-value of 0.072. Such a large negative value indicates a rapid adjustment rate - 82.1% of the deviation is corrected in the subsequent period - but the statistical significance is only marginal at the level of 10%. While this result indicates a strong long-term return for Dubai/Oman, the reliability of the estimate is limited by a p-value just above the traditional 5% threshold. Diagnostic controls provide more clarity on the statistical characteristics of residues in the model. The remaining covariance test yields an F statistic of 543,658 across all three equations, indicating conditional non-heteroskedasticity in the error terms. This suggests that residue variability is not constant over time, violating the assumption of homogeneity and potentially affecting the efficiency and reliability of parameter estimates. This justifies the need for variability modelling techniques that can explicitly account for time-varying variance, such as the VECH-GARCH model. Natural tests of VEC residue also show mixed results. For COBRT and COD, the chi-square statistic is 42.618 with a p-value of 0.000, indicating strong rejection of the null hypothesis of a normally distributed waste distribution. This deviation from normality can lead to biased standard errors and invalid statistical inference if left uncorrected. For WTI, the test statistics are 40,900 with a p-value of 0.100, indicating that the tailings are normally distributed compared to other series.

This relative normality makes WTI results less vulnerable to distribution problems, although positive ECT continues to undermine its interpretation in the context of long-term adaptation. The implications of these results for estimating the VECH-GARCH model are significant. The presence of heterogeneity and unnaturalness in COBRT and COD residues suggests that simple linear models may not adequately capture the volatility dynamics of these series. The VECH-GARCH framework, which allows for time-varying variance and volatility, is essential for accurate modelling of this series. It helps overcome the limitations observed in ECM diagnoses by explicitly modelling conditional covariance and capturing the common movement in fluctuations across oil price parameters. The validity of the VECH model estimate is reinforced by evidence from the ECM suggesting meaningful long-term equilibrium relationships (especially for COBRT and COD) and the need for a more flexible structure to capture the underlying dynamics of variance. This is one of the properties that make Multivariate GARCH models suitable for the study. Therefore, the matrix representation of the results of the Vector Error Conditional Heteroscedasticity (VECH)-GARCH is presented as thus:

$$\mathrm{M} = \begin{bmatrix} 0.090(0.000) & 0.046(0.000) & 0.006(0.003) \\ & 0.023(0.00) & 0.003(0.006) \\ & & 0.001(0.071) \end{bmatrix}$$

$$\mathrm{A} = \begin{bmatrix} 0.105(0.000) & 0.105(0.000) & 0.057(0.000) \\ & 0.104(0.000) & 0.054(0.000) \\ & & 0.026(0.006) \end{bmatrix}$$

$$\mathrm{B} = \begin{bmatrix} 0.659(0.000) & 0.703(0.000) & 0.810(0.000) \\ & 0.748(0.000) & 0.849(0.000) \\ & & 0.965(0.000) \end{bmatrix}$$

Note: The value in the parentheses is the estimated p-value. Alternatively, the model is represented in equation form as:

$$\sigma^2_{i,t} = \mathrm{M} + \mathrm{A1} * \varepsilon^2_{i,t-1} + \mathrm{B1} * \sigma^2_{i,t-1}$$

Variance Equation:

$$\sigma^2_{1,t} = 0.090 + 0.105 * \varepsilon^2_{t-1} + 0.659 * \sigma^2_{1,t-1}$$
$$\sigma^2_{2,t} = 0.024 + 0.104 * \varepsilon^2_{t-1} + 0.748 * \sigma^2_{2,t-1}$$
$$\sigma^2_{3,t} = 0.001 + 0.026 * \varepsilon^2_{t-1} + 0.965 * \sigma^2_{3,t-1}$$

Covariance Equation:

$$\rho_{1,2} = 0.046 + 0.105 * \varepsilon_{1,t-1} * \varepsilon_{2,t-1} + 0.703 * \rho_{1,2,t-1}$$
$$\rho_{1,3} = 0.006 + 0.057 * \varepsilon_{1,t-1} * \varepsilon_{3,t-1} + 0.810 * \rho_{1,3,t-1}$$
$$\rho_{2,3} = 0.003 + 0.054 * \varepsilon_{2,t-1} * \varepsilon_{3,t-1} + 0.849 * \rho_{2,3,t-1}$$

The results of the Vector Error Correction Conditional Heteroscedasticity (VECH)-GARCH model provide insights into stock price volatility, particularly by showing how shocks affect market fluctuations over time. The estimated coefficients in the M, A, and B matrices indicate the nature of volatility persistence and the market's reaction to past disturbances. The M matrix, representing the constant term in the variance equation, contains mostly significant coefficients, with values ranging from 0.001 to 0.090. The significance of most of these values (p-values close to 0.000) suggests that the underlying volatility level is not negligible, though relatively small compared to the time-dependent components. The low value of 0.001 (p=0.071) implies that some components of the conditional variance may be less deterministic and more reliant on external shocks. Compared with previous VECH-GARCH studies, such as Engle and Kroner [5], which also found the constant term to be relatively small, this confirms that market-driven fluctuations often overshadow the intrinsic market volatility. The A matrix, which captures the ARCH effects, measures the short-term impact of past shocks on current volatility. The estimated coefficients range from 0.026 to 0.105, all of which are highly significant (p-values of 0.000). These values indicate that market volatility reacts strongly to recent disturbances, reinforcing the idea of short-term market instability. Similar findings were reported by Bollerslev et al. [2] and Engle and Kroner [5], who found that ARCH parameters were significant in explaining sudden market spikes.

The presence of high ARCH effects in this study suggests that unanticipated events, such as economic announcements or geopolitical shocks, lead to immediate fluctuations in stock prices. The B matrix, capturing the GARCH effects, reflects the long-term persistence of volatility. The coefficients are quite large, ranging from 0.659 to 0.965, and all are statistically significant at the 1% level. The highest value of 0.965 implies that past volatility has a prolonged impact, leading to persistent fluctuations over time. This aligns with previous studies, such as Nelson [10] and Ling and McAleer [9], which found that strong GARCH effects indicate strong volatility clustering, suggesting that market turbulence tends to persist over extended periods. The findings suggest that market participants should anticipate prolonged volatility following major shocks, with implications for risk management strategies. Compared with previous studies on the VECH-GARCH model, the present findings reinforce the well-established pattern of short-term reactivity and long-term persistence in financial markets. The implications are significant for investors and policymakers. High ARCH effects suggest that short-term traders may exploit market shocks, while strong GARCH effects indicate that long-term investors should prepare for extended periods of market uncertainty. Moreover, these findings highlight the need for improved hedging strategies and portfolio diversification, as market fluctuations are not only immediate but also long-lasting (Table 4).

Table 4: Estimation results for portmanteau tests

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	Df
1	22.93649	467.1203	0.2542	467.3803	16
2	46.25668	745.1349	0.1122	745.7044	32
3	59.27973	909.5582	0.0458	910.4025	48
4	91.70694	1083.649	0.1171	1084.882	64
5	100.8715	1197.526	0.0105	1199.076	80
6	113.1738	1266.937	0.0466	1268.719	96
7	122.4939	1313.011	0.0896	1314.973	112
8	155.4524	1377.284	0.1931	1379.534	128
9	172.4273	1430.148	0.0322	1432.663	144
10	192.1867	1482.844	0.0321	1485.654	160
11	204.9934	1525.466	0.0225	1528.539	176
12	223.8200	1571.528	0.0352	1574.910	192

Source: *Researcher's Analysis using Eviews Version 10.*

The Portmanteau test results provide insight into whether residual autocorrelation is present in the multivariate time-series model. Specifically, the test examines the null hypothesis that there is no automatic correlation in the residue until a specific delay order is reached. In the results displayed, both Q statistics and adjusted Q statistics are reported for each delay (1-12), along with p-values and degrees of freedom. A higher p-value (usually greater than 0.05) indicates that the null hypothesis is not rejected, meaning there is no significant subjective correlation at this delay length. At delay 1, the p-values (0.2542 for the adjusted Q case) are higher than the significance level of 5%, indicating that the residues at this delay are not sequentially linked. As the delay order increases, the results fluctuate. For example, in layer 3, the value p drops to 0.0458, suggesting the presence of autocorrelation at the 5% level. However, other delays, such as 4, 6, 7, and 8, show p-values above 0.05 (0.1171, 0.0466, 0.0896, and 0.1931, respectively), indicating mixed evidence of self-correlation across different delays. The p-values are much smaller at layers 5 (0.0105), 9 (0.0322), 10 (0.0321), and 11 (0.0225), suggesting intermittent self-correlation in system residues at these specified delay lengths. However, although some delays are significant, the majority do not consistently reject the null hypothesis across all periods. Therefore, the general implication is that residual autocorrelation is systematically absent in the model, suggesting an acceptable level of model adequacy with respect to residual behaviour. These results support the reliability of the VECH-GARCH model used in the study.

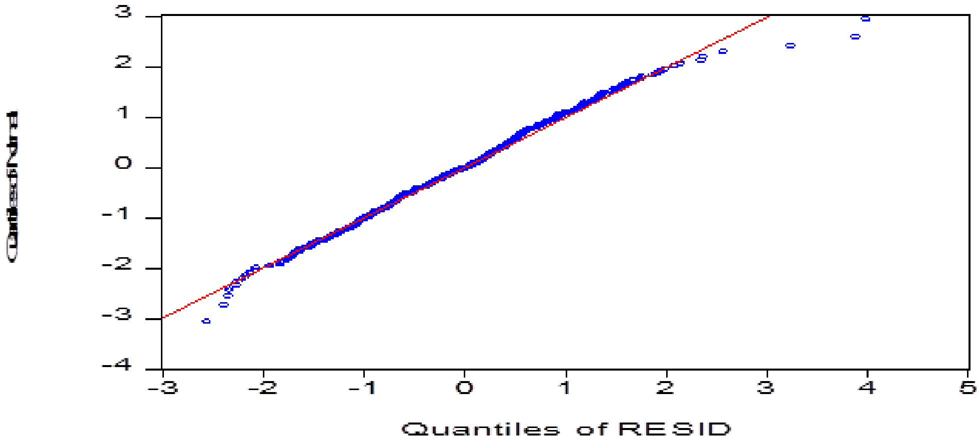

Figure 3: QQ plot on the VECH-GARCH model to determine the adequacy of the model

The accompanying QQ chart (Figure 3) serves as a visual diagnosis to assess the nature of the remains. If the residues are normally distributed, the points in the QQ chart will closely follow the 45° reference line. Deviation from this line means unnaturalness, which can affect inference based on standard statistical tests. Therefore, the Portmanteau test and the QQ chart together provide strong evidence of model adequacy, helping verify the suitability of the VECH-GARCH framework for capturing the conditional heterogeneity and dynamic behaviour of the underlying data series.

4. Discussion of Results

Findings from the Error Correction Model (ECM) provide valuable insights into the dynamics of crude oil price standards – Brent, West Texas Intermediate, and Dubai/Oman– in response to short-term disruptions and their tendencies to return to long-term equilibrium. ECM coefficients represent how quickly each series corrects deviations from equilibrium. For Brent Blend, the Error Correction Term (ECT) of -0.507 is statistically significant at the level of 5% (p = 0.03), suggesting that Brent is returning quickly, with about 50.7% correction of the imbalance in the next period. This is in line with studies such as Aloui and Jammazi [1], who found that Brent crude prices rapidly adapt to global shocks due to their status as a global benchmark. Conversely, the West Texas Intermediate is 0.658 (r = 0.060), which shows that the estimate is not only statistically insignificant at the 5% level but also incorrectly signed, indicating divergence rather than convergence. This means that West Texas Intermediate does not respond in a theoretically consistent way to imbalance. This finding may reflect regional distortions or infrastructure bottlenecks in the U.S. oil market, consistent with Kilian [8], who noted that West Texas Intermediate could be split locally from global crude markets under certain conditions. Also, for crude oil prices in Dubai/Oman, the ECT is -0.821 (p = 0.072), indicating a very fast adjustment (82.1%) of the balance, albeit only significant at the 10% level.

The large volume of the adjustment supports evidence from studies such as Hammoudeh et al. [7], which highlight the high sensitivity of Middle Eastern crude oil standards to global demand and geopolitical conditions. Diagnostic tests provide additional clues regarding the adequacy of the model. Residual invariance was detected across the three equations (F = 543.658), indicating variable volatility over time, violating the assumption of constant variance. Also, tests of the normality of the residuals confirm the presence of abnormal distributions in Brent and Dubai/Oman (p = 0.000), whereas WTI shows only an approximate normal distribution (p = 0.100). These findings support the need for GARCH-type models to capture better the stylised features of financial time series, including volatility aggregation and heavy tails. The VECH-GARCH model estimate reveals deeper insights into the transmission of volatility. The fixed matrix (M) captures the underlying volatility levels. It is relatively small, consistent with Engle and Kroner [5], suggesting that the main source of volatility arises from shocks and stability effects rather than constant volatility levels. The ARCH(A) matrix, with significant coefficients ranging from 0.026 to 0.105, indicates a strong short-term response to shocks. This aligns with the findings of Bollerslev et al. [2], who asserted that energy markets respond quickly to unexpected macroeconomic and geopolitical developments. More importantly, the GARCH(B) coefficients, ranging from 0.659 to 0.965, indicate strong volatility persistence, confirming the findings of Nelson

[10] and Ling and McAleer [9]. These significant values suggest that shocks have long-term effects, causing volatility to persist over long periods – a critical pattern for investors, policymakers, and risk managers.

The results of the Portmanteau test further confirm the model's structural correctness. While some delays (e.g., 3, 5, 9, 10, and 11) show autocorrelation at the 5% level ($p < 0.05$), most delays indicate no significant autocorrelation in the residues. This supports the adequacy of the VECH-GARCH model for sequential disengagement, a prerequisite for modelling strong multivariate time series. The Q-Q scheme confirms these results by assessing the distributional properties of the residues. Deviations from the 45-degree baseline highlight the abnormality, especially for crude oil prices in Brent and Dubai/Oman, supporting the need for conditionally heterogeneous models. The combination of ECM and VECH-GARCH diagnostics suggests that Brent and Dubai/Oman exhibit meaningful long-term equilibrium behaviour and predictable volatility patterns. In contrast, West Texas Intermediate behaviour is less stable and is likely to be affected by local supply and demand constraints. Compared to previous empirical work, this study reaffirms the asymmetric and region-specific nature of crude oil volatility, consistent with Ewing and Malik [6], who found structural differences in volatility dynamics between oil parameters. Furthermore, the results expand the literature by integrating the speed of equilibrium adjustment and volatility extension into a unified framework, emphasising both short-term market interaction and long-term stability.

5. Conclusion

The study concluded that crude oil price standards show distinct adjustment mechanisms and volatility dynamics, reinforcing the heterogeneous nature of global oil markets. Brent crude and Dubai/Oman showed a large and theoretically consistent return to long-term equilibrium after short-term deviations, with Brent retracement correcting 50.7% and pushing 82.1% of the previous imbalance. These results underscore the dominant roles of Brent and Dubai/Oman in determining global oil prices, as their prices adjust quickly to reflect market fundamentals and shifting global demand. However, the West Texas Intermediate index shows a positive, statistically insignificant error correction term, indicating a deviation from equilibrium that may stem from regional supply bottlenecks, infrastructure constraints, or local market segmentation. This divergence challenges the effectiveness of West Texas Intermediate as a responsive benchmark in international price-fixing. Volatility diagnostics from the VECH-GARCH model also reveal that while baseline volatility is relatively modest, short-term shocks (ARCH effects) and long-term stability (GARCH effects) are strongly evident. These results suggest that market volatility is strongly influenced by both direct events and prolonged uncertainty, consistent with Nelson's [10] empirical results.

The property of volatility's long memory (B values up to 0.965) indicates that once shocks occur, their effects persist in the market, affecting investor behaviour and policy expectations over time. The implications of these findings for determining the crude oil market are multifaceted. First, the rapid adjustment rates of Brent crude and Dubai/Oman reinforce their centrality in global pricing and confirm their reliability in reflecting global market fundamentals. Policymakers and market analysts can rely more confidently on these criteria for decision-making and policy formulation. Second, West Texas Intermediate's failure to comply with the dynamics of equilibrium adjustment points to the need to reassess its role in global oil pricing, particularly during periods of regional market pressure. Third, the continuous volatility patterns revealed by the VECH-GARCH model highlight the importance of integrating volatility modelling into risk assessment and forecasting. For investors, these dynamics underscore the need for strong hedging and diversification strategies. In contrast, for regulators and policymakers, the results underscore the need to enhance transparency, infrastructure development, and market integration, particularly for benchmarks such as West Texas Intermediate. The study contributes to understanding oil price behaviour by illustrating how adjustment speeds and continued volatility vary across reference criteria. It emphasises that global crude oil markets are characterised by asymmetrical and structural inequalities that affect price formation and risk exposure, necessitating divergent approaches to forecasting, investment, and political interventions.

Acknowledgment: The authors express sincere gratitude to Rivers State University for providing the academic environment that supported this work.

Data Availability Statement: All data generated or analyzed during this study are available from the authors upon reasonable request.

Funding Statement: This research was carried out entirely through the author's own efforts and received no external funding, grants, or financial support.

Conflicts of Interest Statement: The authors confirm that there are no conflicts of interest associated with this research.

Ethics and Consent Statement: The research was conducted in accordance with established ethical principles, ensuring the protection and respect of all participants.

References

1. C. Aloui and R. Jammazi, “The effects of crude oil shocks on stock market shifts behaviour: A regime switching approach,” *Energy Economics*, vol. 31, no. 5, pp. 789–799, 2009.
2. T. Bollerslev, R. F. Engle, and J. M. Wooldridge, “A capital asset pricing model with time-varying covariances,” *Journal of Political Economy*, vol. 96, no. 1, pp. 116–131, 1988.
3. Z. D. Deebom and I. D. Essi, “Modeling price volatility of Nigerian crude oil markets using GARCH model: 1987–2017,” *International Journal of Applied Science and Mathematical Theory*, vol. 8, no. 4, pp. 23–48, 2017.
4. Z. D. Deebom and G. L. Tuaneh, “Modeling exchange rate and Nigerian deposit money market dynamics using trivariate form of multivariate GARCH model,” *Asian Journal of Economics, Business and Accounting*, vol. 10, no. 2, pp. 1–18, 2019.
5. R. F. Engle and K. F. Kroner, “Multivariate simultaneous generalized ARCH,” *Econometric Theory*, vol. 11, no. 1, pp. 122–150, 1995.
6. B. T. Ewing and F. Malik, “Volatility transmission between oil prices and the stock market under structural breaks,” *Global Finance Journal*, vol. 29, no. 2, pp. 12–23, 2016.
7. S. Hammoudeh, S. Dibooglu, and E. Aleisa, “Relationships among U.S. oil prices and oil industry equity indices,” *International Review of Economics & Finance*, vol. 13, no. 4, pp. 427–453, 2004.
8. L. Kilian, “Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market,” *American Economic Review*, vol. 99, no. 3, pp. 1053–1069, 2009.
9. S. Ling and M. McAleer, “Stationarity and the existence of moments of a family of GARCH processes,” *Journal of Econometrics*, vol. 106, no. 1, pp. 109–117, 2002.
10. D. B. Nelson, “Conditional heteroskedasticity in asset returns: A new approach,” *Journal of the Econometric Society*, vol. 59, no. 2, pp. 347–370, 1991.
11. R. S. Pindyck, “Volatility and commodity price dynamics,” *The Journal of Futures Markets*, vol. 24, no. 11, pp. 1029-1047, 2004.
12. R. Su, J. Du, F. Shahzad, and X. Long, “Unveiling the effect of mean and volatility spillover between the United States economic policy uncertainty and WTI crude oil price,” *Sustainability*, vol. 12, no. 16, p. 6662, 2020.
13. R. Smyth and P. K. Narayan, “Applied econometrics and implications for energy economics research,” *Energy Economics*, vol. 50, no. 7, pp. 351-358, 2015.
14. Z. Liu, R. L. Peach, F. Laumann, S. Vallejo Mengod, and M. Barahona, “Kernel-based joint independence tests for multivariate stationary and non-stationary time series,” *arXiv preprint*, arXiv:2305.08529, 2023. Available: https://arxiv.org/abs/2305.08529 [Accessed by 17/11/2024].
15. P. S. Daewii, N. B. John, D. Iyai, D. Z. Dum, and O. E. Adedayo, “The impact of exogenous variables on stock market volatility: A GARCH-X model approach,” *AVE Trends in Intelligent Management Letters*, vol. 1, no. 1, pp. 28–37, 2025.
16. S. S. Kumar, S. P. Kumar, S. Singh, and A. S. Kumar, “Dynamics of the evolving financial sector: Challenges and opportunities for Indian digital banks,” *AVE Trends in Intelligent Technoprise Letters*, vol. 1, no. 1, pp. 30–39, 2024.